

Mathematical Girls - Fermat's Last Theorem

C05

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On a unit circle whose center is the origin, do infinitely many rational points exist?

FIRST WE REFORMULATE THE QUESTION

$$x^2 + y^2 = 1$$

WE WILL SUPPOSE (x, y) ARE THE POINTS OF THE COORDINATE PLANE.

THE EQUATION REPRESENTS A CIRCLE WITH CENTER AT THE ORIGIN AND RADIUS ONE WILL BE

THE PROPOSITION THAT INFINITELY MANY RATIONAL POINTS EXIST ON THIS CIRCLE IS

EQUIVALENT TO THE PROPOSITION THAT THE EQUATION $x^2 + y^2 = 1$

HAS INFINITELY MANY RATIONAL SOLUTIONS.



WE
DRAW THE
STRAIGHT
LINE l WITH
SLOPE 1.

LINE NOW
THROUGH
POINT $P(-1, 0)$
WHICH IS ON
THE CIRCLE-
PERIPHERY.

We cut the unit circle
with the straight line l .

$$y = tx + t$$

SINCE ITS
SLOPE IS 1
AND IT PASSES
THROUGH THE
POINT $T(0, 1)$, THE
EQUATION OF
THE STRAIGHT
LINE l WILL BE
LIKE THIS.

LET'S
CALL THAT
POINT Q.



IF WE
EXTEND THE
STRAIGHT LINE
 l TOWARDS
POINT Q,

THERE MUST
BE ONE OTHER
POINT (OTHER
THAN T) WHERE THE
STRAIGHT LINE l
AND THE CIRCLE
INTERSECT.

$$\begin{cases} x^2 + y^2 = 1 & \text{equation of the circle} \\ y = tx + t & \text{equation of the straight line } L \end{cases}$$

TO EXPRESS
Q IN TERMS OF
t, WE CAN USE
THE FOLLOWING
SIMULTANEOUS
EQUATIONS.

PAIRS OF REAL
SOLUTIONS OF SIMUL-
TANEOUS EQUATIONS
CORRESPOND TO POINTS
OF INTERSECTION IN THE
GEOMETRY REPRESENTING
THE SOLUTIONS.

$$x^2 + y^2 = 1 \quad \text{equation of the circle}$$

$$x^2 + (tx + t)^2 = 1 \quad \begin{array}{l} \text{substitute in} \\ y = tx + t \end{array}$$

$$x^2 + t^2 x^2 + 2tx + t^2 = 1 \quad \text{expand}$$

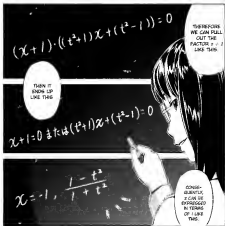
$$x^2 + t^2 x^2 + 2tx + t^2 - 1 = 0 \quad \text{move the 1}$$

$$(t^2 + 1)x^2 + 2tx + t^2 - 1 = 0 \quad \text{collect } x^2 \text{ terms}$$

THIS
WILL BE A
QUADRATIC
EQUATION
IN x.

SINCE
 $t^2 + 1 \neq 0$,



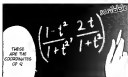




DO YOU UNDERSTAND SO FART?

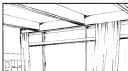


THEN, WE GET
 $x = \frac{1-t^2}{1+t^2}$ AND
 $y = \frac{2t}{1+t^2}$.



THESE
 ARE THE
 COORDINATES
 OF Q

$$\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2} \right)$$



IF WE USE
 THE EQUATION
 OF THE STRAIGHT
 LINE $y = x + 1$ THEN
 IT CAN ALSO BE
 EXPRESSED IN
 TERMS OF t .

$$y = x + 1$$

$$\begin{aligned} &= t \left(\frac{1-t^2}{1+t^2} \right) + 1 \\ &= \frac{t(1-t^2)}{1+t^2} + 1 \\ &= \frac{t(1-t^2) + t(1+t^2)}{1+t^2} \\ &= \frac{t(1-t^2) + t(1+t^2)}{1+t^2} \\ &= \frac{2t}{1+t^2} \end{aligned}$$

SINCE $(x, y) = \left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2} \right)$
 IS NOT POINT Q, WE
 WILL ONLY FOLLOW
 $y = \frac{2t}{1+t^2}$



NOW PAY
ATTENTION TO
POINT T WHICH
IS LOCATED ON
THE Y-AXIS



WILL
THEM

IN THE
FIRST PLACE,
I WAS THINKING
WHY THERE
SOMETHING
INFINITE

THAT CAN
BE APPROXIMATED
BY ONE-TO-ONE
CORRESPONDENCE
TO THE RATIONAL
POINTS ON THE
CIRCUMFERENCE
OF THE CIRCLE



IF POINT T IS
A RATIONAL POINT
LOCATED ON THE
Y-AXIS, THEN POINT
Q WILL BE A
RATIONAL POINT
AS WELL

WHICH
MEANS—

THE
COORDINATES
OF POINT Q CAN
BE CONSTRUCTED BY
APPLYING THE FOUR
BASIC ARITHMETIC
OPERATIONS TO THE
COORDINATES
OF POINT T,
NAMESLY (1)...



BECAUSE A
RATIONAL POINT CAN
BE OBTAINED BY
APPLYING THE FOUR
BASIC ARITHMETIC
OPERATIONS TO
A RATIONAL
NUMBER

WILL BE
A RATIONAL
NUMBER AS
WELL

These
movements
in them-
selves are

WE CAN
MOVE POINT P
EASILY TO ANY
OF THE INFINITE
RATIONAL POINTS
LOCATED ON
THE LINE.

AND AS
WE CHANGE
POINT P, POINT Q
CHANGES
AS WELL.

dignified.

and beautiful.

—At
any rate.

there are
no pointless
movements

when
Minako-san is
doing maths.





SO A UNIT
CIRCLE COMES
OUT OF THE
PYTHAGOREAN
THEOREM!



$(x, y) = (a/c, b/c)$
IS A SOLUTION OF
 $x^2 + y^2 = 1$

AND



$(a/c, b/c)$
CORRESPONDING
TO THEM.

FOR
DIFFERENT
PRIMITIVE
PYTHAGOREAN
TRIPLES, THERE
ARE DIFFERENT
RATIONAL
POINTS



I HOPED YOU
TO SAY THAT THE
RATIONAL POINTS
ON THE CIRCUMFER-
ENCE OF THE UNIT
CIRCLE COME OUT,
THOUGH.



ON A
UNIT CIRCLE
WHOSE CENTER
IS THE ORIGIN, SO
INFINITELY MANY
RATIONAL POINTS
EXIST** ARE
EQUIVALENT TO
EACH OTHER.

ARE THERE
INFINITELY MANY
PRIMITIVE
PYTHAGOREAN
TRIPLES** AND

The correspondence between the unit
circle and the Pythagorean triples.





**I DIDN'T
NOTICE...**



ON
BIRCHER-MAN'S
CARD: A RELATION
OF SUFFICIENT
NUMBERS WAS
WRITTEN

On a new circle whose
center is the origin, do
infinitely many colored
points exist?



ON
TYPEN-CARD'S
CARD: A RELATION
OF SUFFICIENT WAS
WRITTEN

All these infinitely
many primitive
Pythagorean triples?



**EVEN THOUGH
I SAW THEM...**

**EVEN
THOUGH I
SAW BOTH
CARDS...**





ALGEBRA AND
GEOMETRY—

SURROUNDED
BY THAT
BITTERNESS,
REFRESHING
SMELL...



MY HEART
MYSTERIOUSLY
CALMED DOWN.

PERHAPS
MURAKI-SENSEI
WANTED TO SHOW
US THOSE TWO
WORLDS.



TWO
WORLDS...

THOSE WHO JOURNEY
THE STARS -- THOSE
WHO DREAM THE
CONSTELLATIONS.

WHEN ONE IS
AT-ONE



SAY TETRA,
WHY ARE YOU
ONLY WITH
THOSE GUYS?



WELL, AS FOR
THAT BECAUSE YOU
ALREADY COMED
TO HIGH SCHOOL, I
KNOW THAT YOU LIKE
GIRLS, BUT STILL.



ACTING FRIENDLY
EVEN WITH A GIRL
WHO'S TAKING
AWAY SCENE??

SERIOUS.

WHY ARE YOU





THESE ARE
FOURBOARDS THAT
CAN BE BUILT ON
TOP OF THAT SORT
OF WINTERBOSS
RELATIONSHIP,
ABOUT THERE?

SO, JUST LIKE
MIRRORE-TUN AND
TERRA-COATS.



THERE ARE
ALL SORTS OF
PEOPLE IN OUR
CLUB TOO THERE
AND ALL SORTS
OF PEOPLE IN
OUR CLUB TOO.



WOOSH

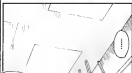
—TODAY
YOU SEE.

SQUEEZE



WE
ARE STILL
BLOSSOMING
HIGH-SCHOOL
GIRLS.

WHETHER IT'S
STUDY OR LOVE,
THE DEVELOPMENT
(MORATORUM) OF
OUR LIVES IS IN
FULL BLOOM!



Diagrams and equations.

In mathematics, there are times when we use equations or deal with diagrams.

For example, as Makuu can write on the blackboard, "A circle whose center is the origin and whose radius equals 1" can be expressed by the equation " $x^2+y^2=1$ ". That's called the equation of the circle. If we assume the blackboard is a coordinate plane, a pair of numbers x and y , namely (x, y) , will express one point on the blackboard. Let's suppose we gather all the points (x, y) that fulfil the equation of the circle $x^2+y^2=1$ and then mark them on the blackboard. If we do that, these points will collectively make a circle.

Makuu can also used an equation for a straight line, namely " $y=2x+1$ ".

The ordered pairs (x, y) that fulfil both the straight line equation and the circle equation are the points where the circle and the straight line cross each other, in other words, points of intersection. That's to say, by solving the simultaneous equations, we can obtain the points of intersection.



数学ガール

Mathematics Girl: Fermat's Last Theorem

フェルマーの最終定理